Bulgarian Academy of Sciences
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Department of Physics and Astronomy

Oblique-Basis Shell Model Method

- **The idea:** to combine different shell model bases:
  - m-scheme spherical shell-model
  - SU(3) symmetrv based shell-model
- **Developers …** Louisiana State University LSU
  - **Vesselin Gueorguiev**
  - **Jerry Draayer**
  - **Erich Ormand**
  - **Calvin Johnson**

$H\Psi = E\Psi$

m-scheme states

+

SU(3) states

$(H - Eg)\Psi=0$

Oblique-SMC

CONSEJO SUPERIOR DE INVESTIGACIONES CIENTIFICAS
INSTITUTO DE ESTRUCTURA DE LA MATERIA

Physics & Advanced Technologies N Division

Lawrence Livermore National Laboratory

Problems that we understand well

exactly solvable (symmetry at play)

perturbative regime

$$H = H_0$$

$$H = H_0 + V$$

$$H = H_1(\alpha) + H_2(\beta) + \ldots$$

What about **more than one** *exactly solvable part* beyond **the perturbative regime?**

Transition from **phase one**

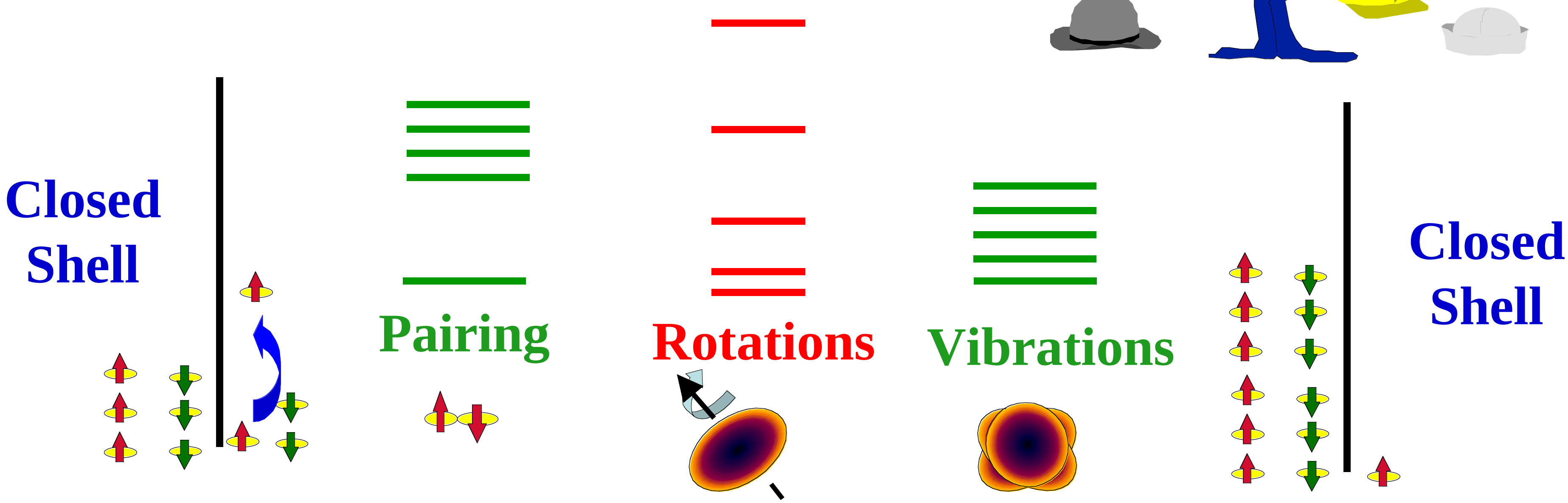
to **phase two** should occur.

Can two or more different sets of basis states be used to gain a deeper understanding of the basic physics?

The Challenge in Nuclei...

Nuclei display unique characteristics:

- **Single-particle Features**
- **Pairing Correlations**
- **Deformation/Rotations**



Outline of the Talk

- **Introduction…**
- **General motivation…**

  (exactly solvable ⇔ symmetry, small perturbation, …)

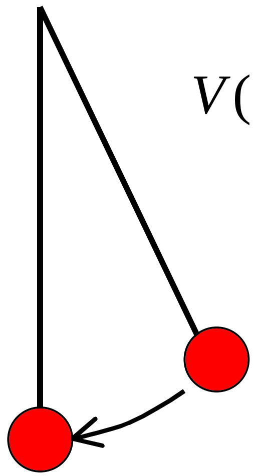
- **Two-mode toy model: the harmonic oscillator in a box**

- **Real nuclei:**

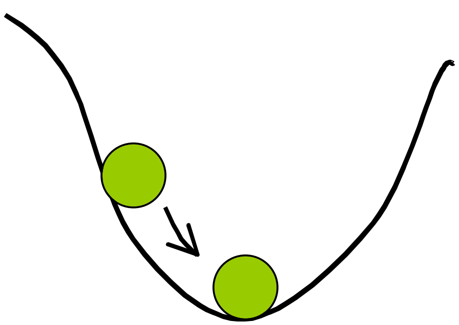
  interplay b/w single-particle & collective excitations

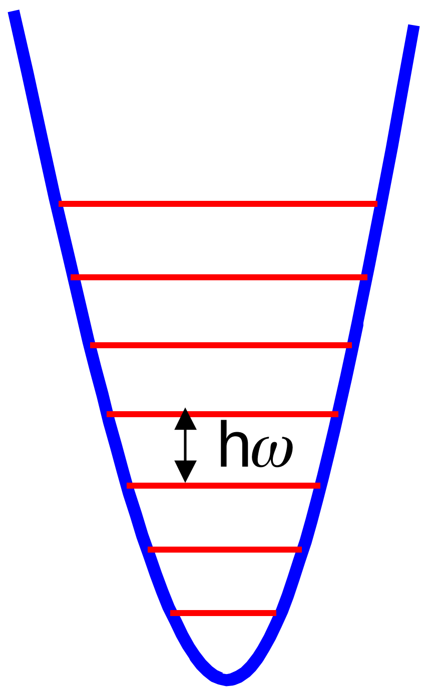
- **Conclusions**

Harmonic Oscillations for System Near Equilibrium



$$V(x) = V_0 + \frac{1}{2} m\omega^2 x^2$$





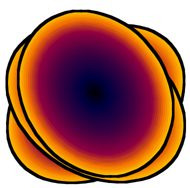
A quantized 1D oscillator is an **exactly solvable** system with **equally spaced** levels:

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

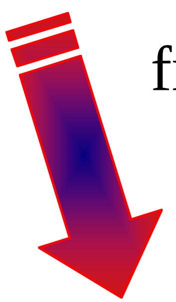
$$\Delta E = \hbar\omega$$

Particle in 1D Box

$$V(x) = \begin{cases} 0 & \text{if } |x| < a \\ +\infty & \text{if } |x| \geq a \end{cases}$$

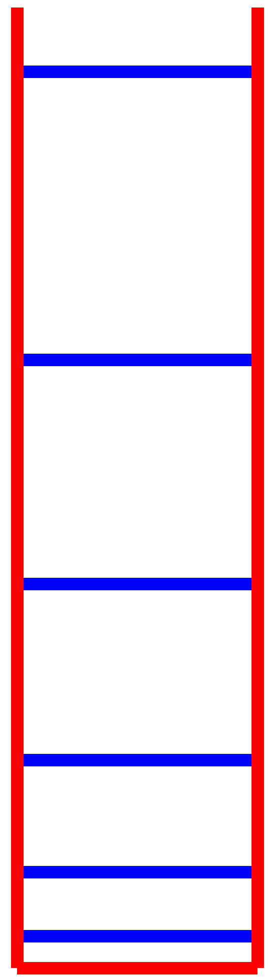


Finite Volume Confinement



from a 3D bag
to
a 1D box

-a **a**



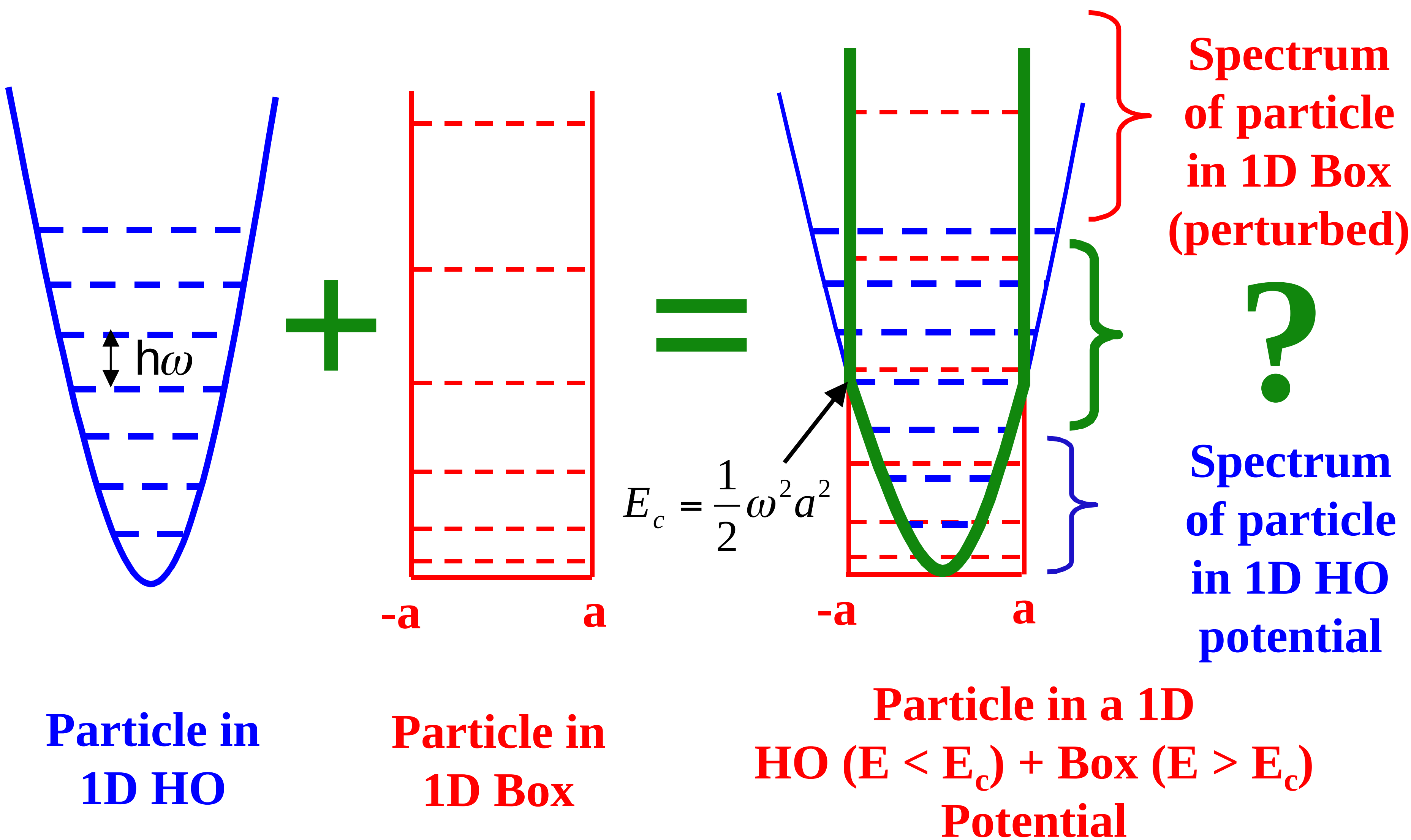
-a **a**

A quantized 1D box is an **exactly solvable** system with discrete energy levels with **increasing spacing**:

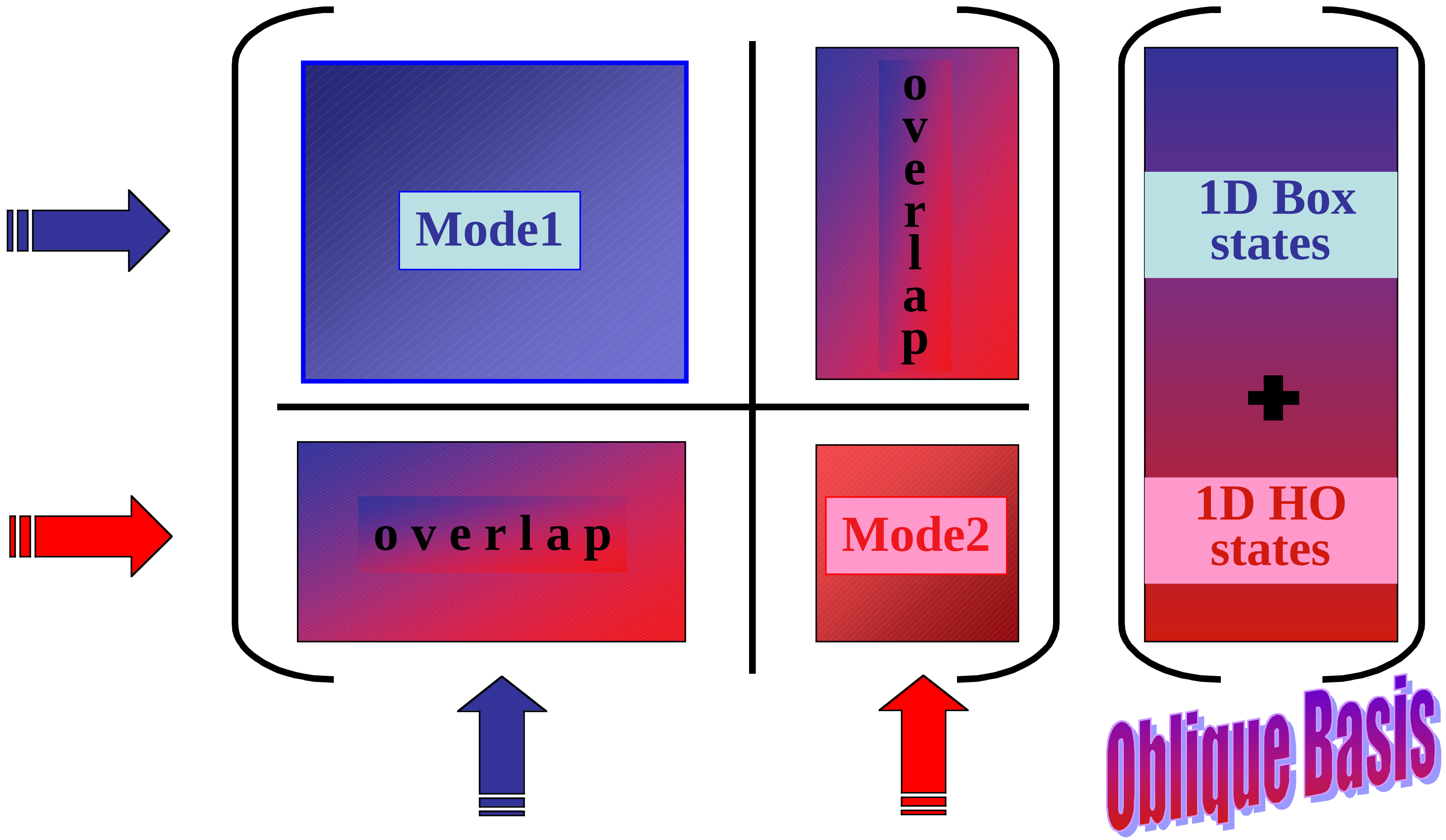
$$E_n = \frac{1}{2}\left(\frac{\pi\hbar}{2a}\right)^2 (n+1)^2$$

$$\Delta E = \frac{1}{2}\left(\frac{\pi\hbar}{2a}\right)^2 (2n+1)$$

Two-Mode Toy System



Hamiltonian Matrix in Oblique Basis

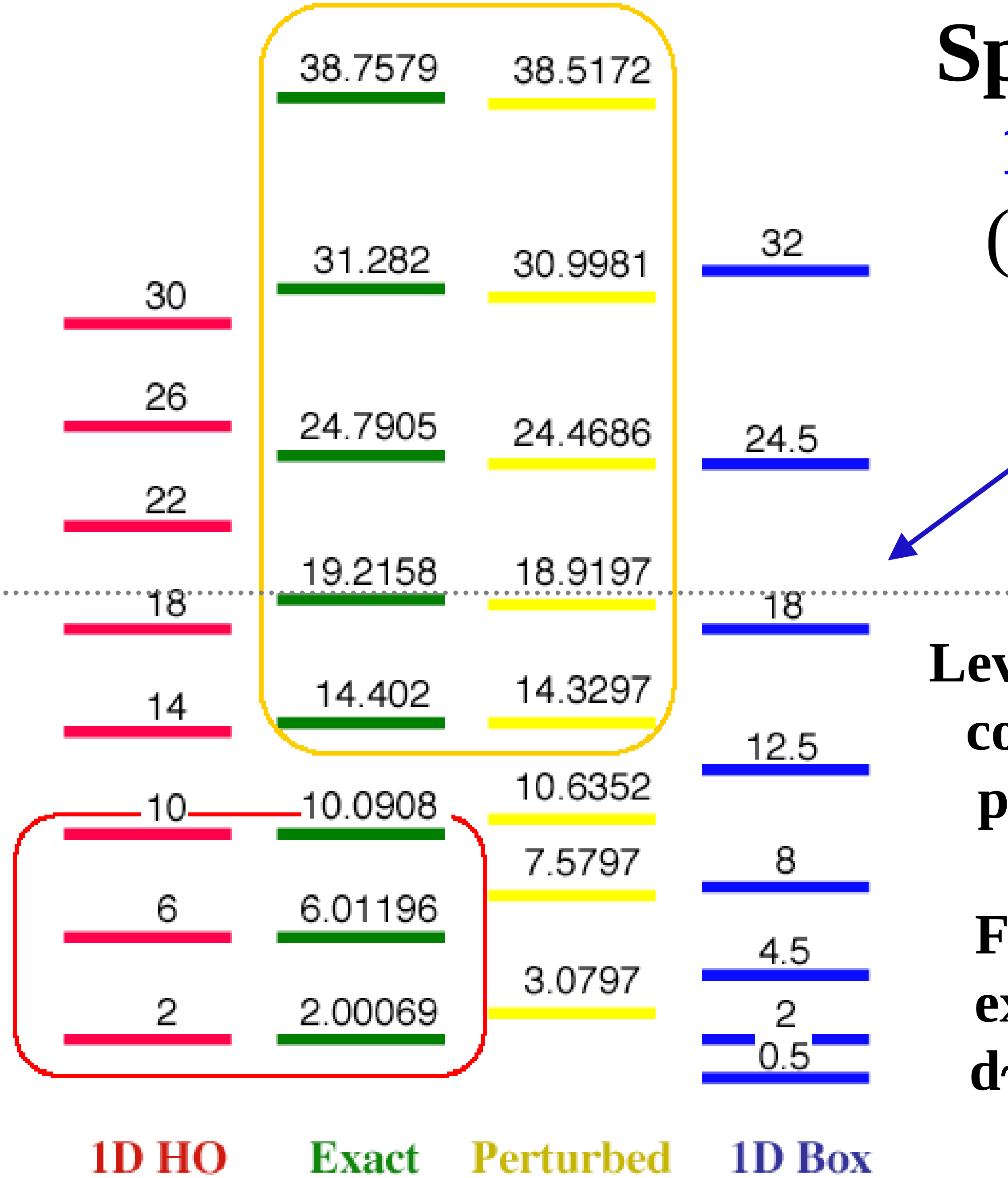


Spectral Structure

1D Box + 1D HO

(m=1, a=π/2, ω=4)

$$E_c = \frac{1}{2}\omega^2 a^2 = 19.74$$



Levels above ~10 (below ~ 4) coincide with the levels of particle in 1D Box (HO).

First 8 levels converge to exact results (~0.01%) at d~4+10=14 oblique basis. (18 standard basis)

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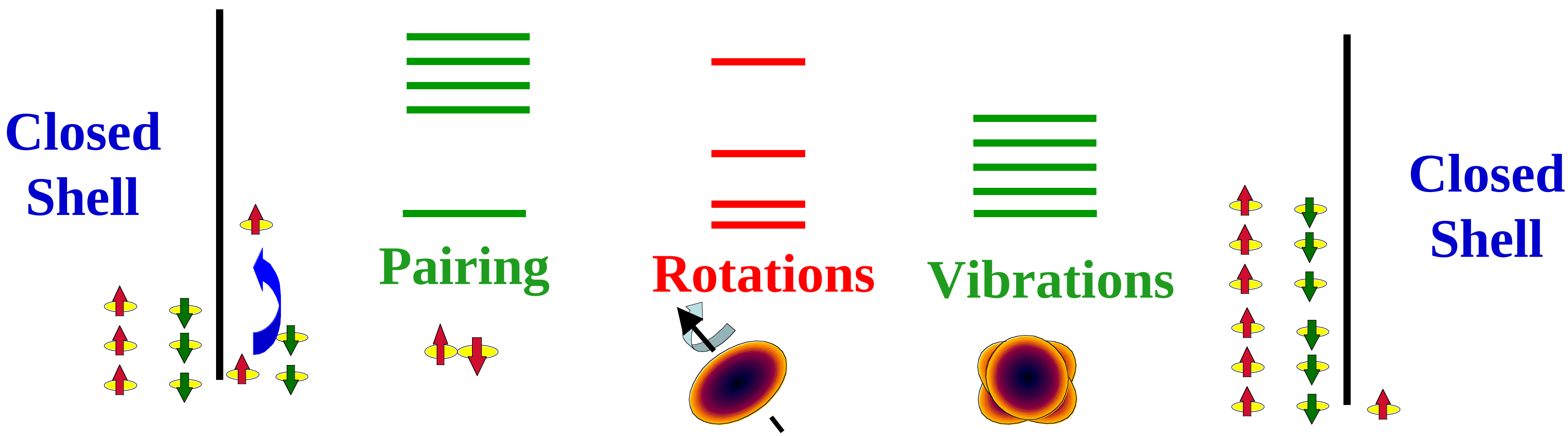


Nuclear Shell-Model Hamiltonian

$$H = \sum_i \varepsilon_i a_i^+ a_i + \sum_{i,j,k,l} V_{ijkl} a_i^+ a_j^+ a_k a_l \;=\; \sum_i \varepsilon_i N_i + \chi Q \cdot Q + U_{\text{residual}}$$

where a_i^+ and a_i are fermion creation and annihilation operators,

ε_i and V_{ijkl} are real and $V_{ijkl} = V_{klij} = -V_{jikl} = -V_{ijlk}$

- Spherical shell-model basis states are eigenstates of the one-body part of the Hamiltonian - **single-particle states**.

- The two-body part of the Hamiltonian H is dominated by the **quadrupole-quadrupole interaction** Q·Q ~ C_2 of SU(3).

- SU(3) basis states - **collective states** - are eigenstates of H for degenerate single particle energies **ε** and a pure Q·Q interaction.

Eigenvalue Problem in an Oblique Basis

- Spherical basis states $\mathbf{e_i}$
- SU(3) basis states $\mathbf{E_\alpha}$

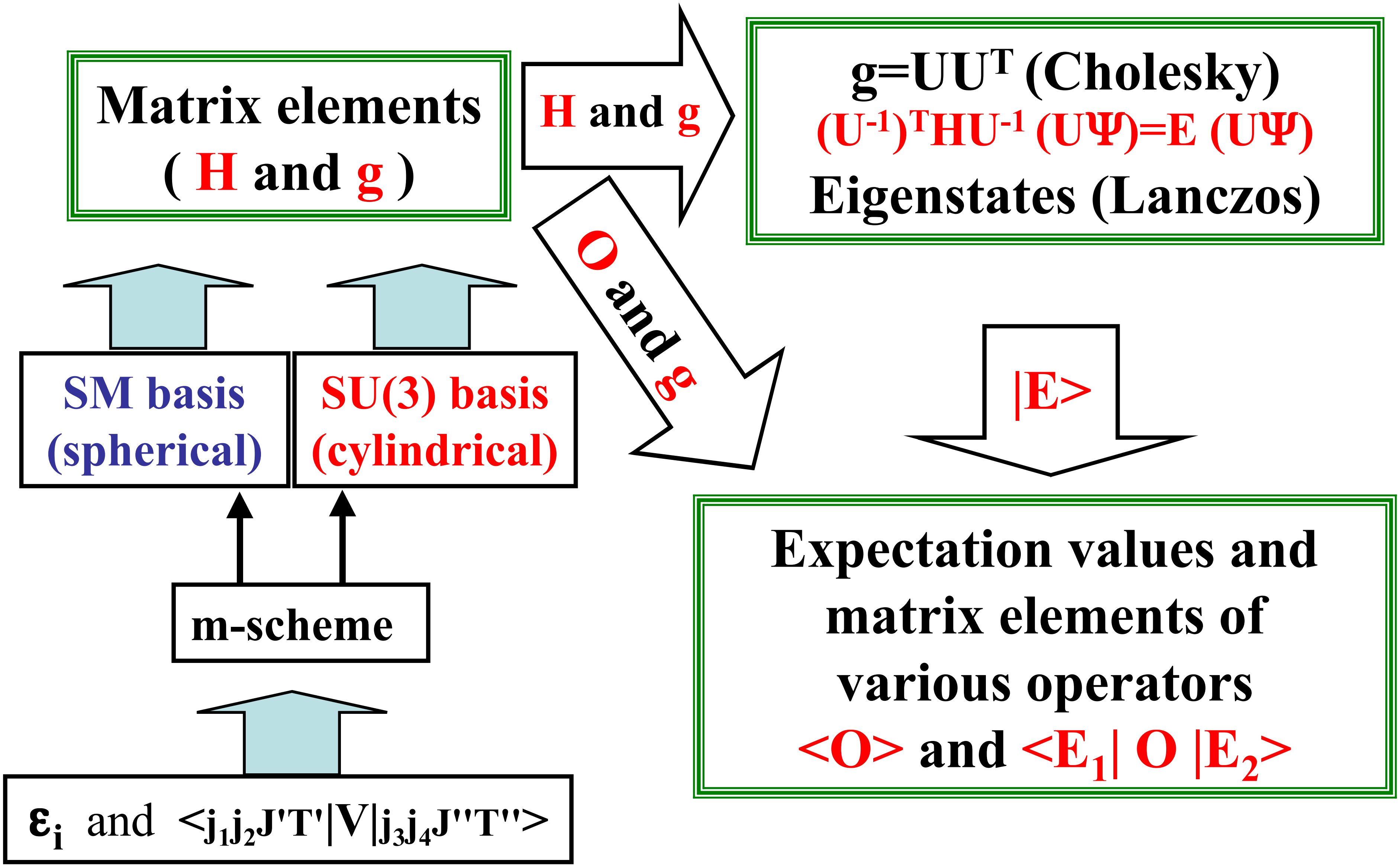
- Overlap matrix $\mathbf{g}$

$$\hat{g} = \begin{pmatrix} \langle e_i | e_j \rangle & \langle e_i | E_\beta \rangle \\ \langle E_\alpha | e_j \rangle & \langle E_\alpha | E_\beta \rangle \end{pmatrix} = \begin{pmatrix} 1 & \mu \\ \mu^+ & 1 \end{pmatrix}$$

- The eigenvalue problem

$$H\psi = E\psi \quad \Rightarrow \quad \hat{H} \cdot \hat{\psi} = E\, \hat{g} \cdot \hat{\psi}$$

Current Evaluation Steps

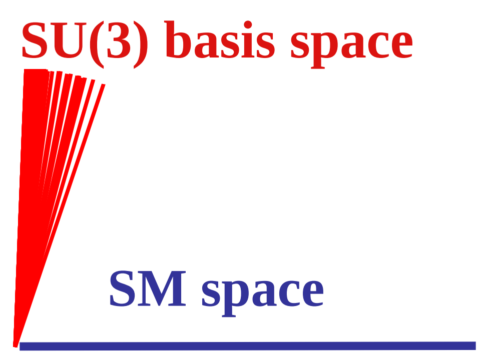


Example of an Oblique Basis Calculation: ^{24}Mg

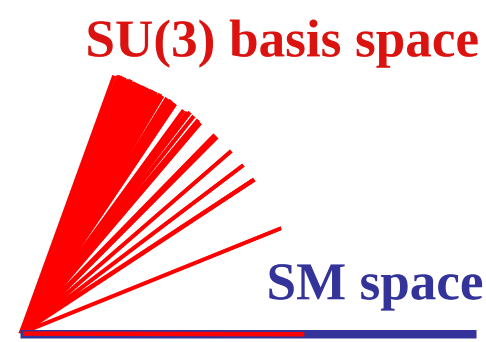
We use the **Wildenthal USD interaction** and denote the **spherical basis** by SM(#) where # is the number of nucleons outside the $d_{5/2}$ shell, the **SU(3) basis** consists of the leading irrep (8,4) and the next to the leading irrep, (9,2).

Model Space	SU3 (8,4)	SU3+ (8,4) & (9,2)	GT100	SM(0)	SM(1)	SM(2)	SM(4)	Full
Dimension (m-scheme)	**23**	**128**	**500**	**29**	**449**	**2829**	**18290**	**28503**
%	**0.08**	**0.45**	**1.75**	**0.10**	**1.57**	**9.92**	**64.17**	**100**

Visualizing the SU(3) space with respect to the SM space using the naturally induced basis in the SU(3) space.

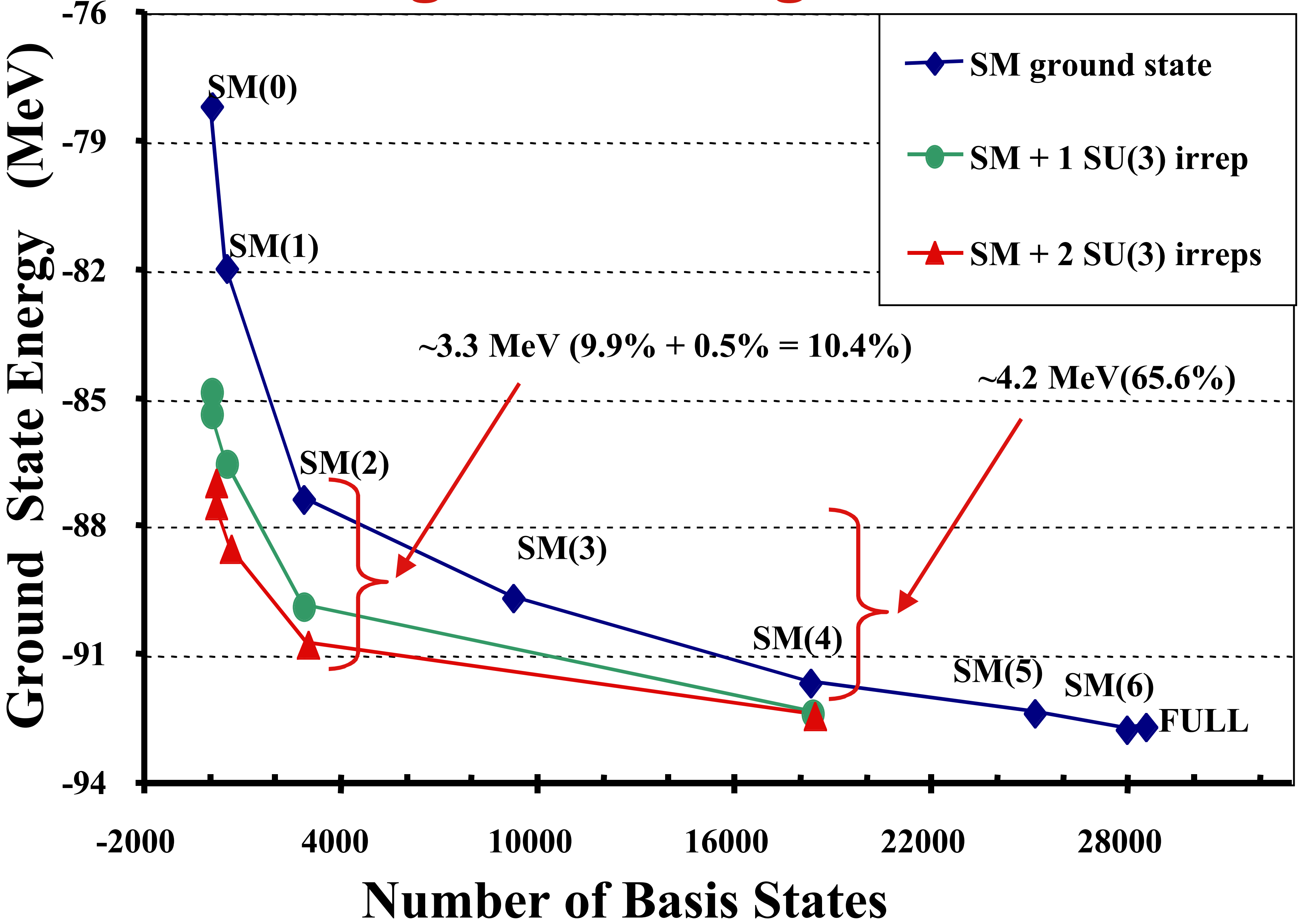


SM(2) & SU3+

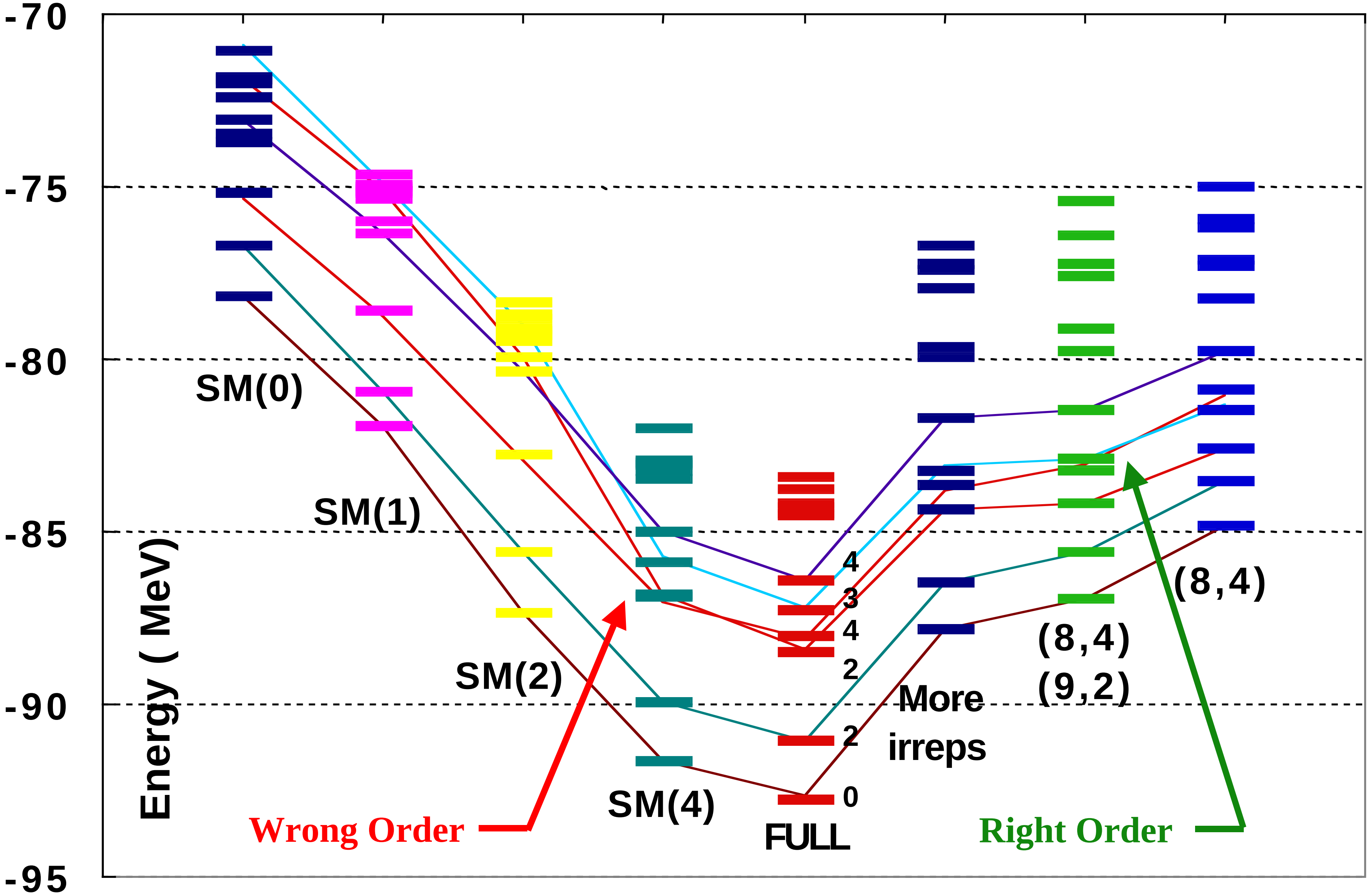


SM(4) & SU3+

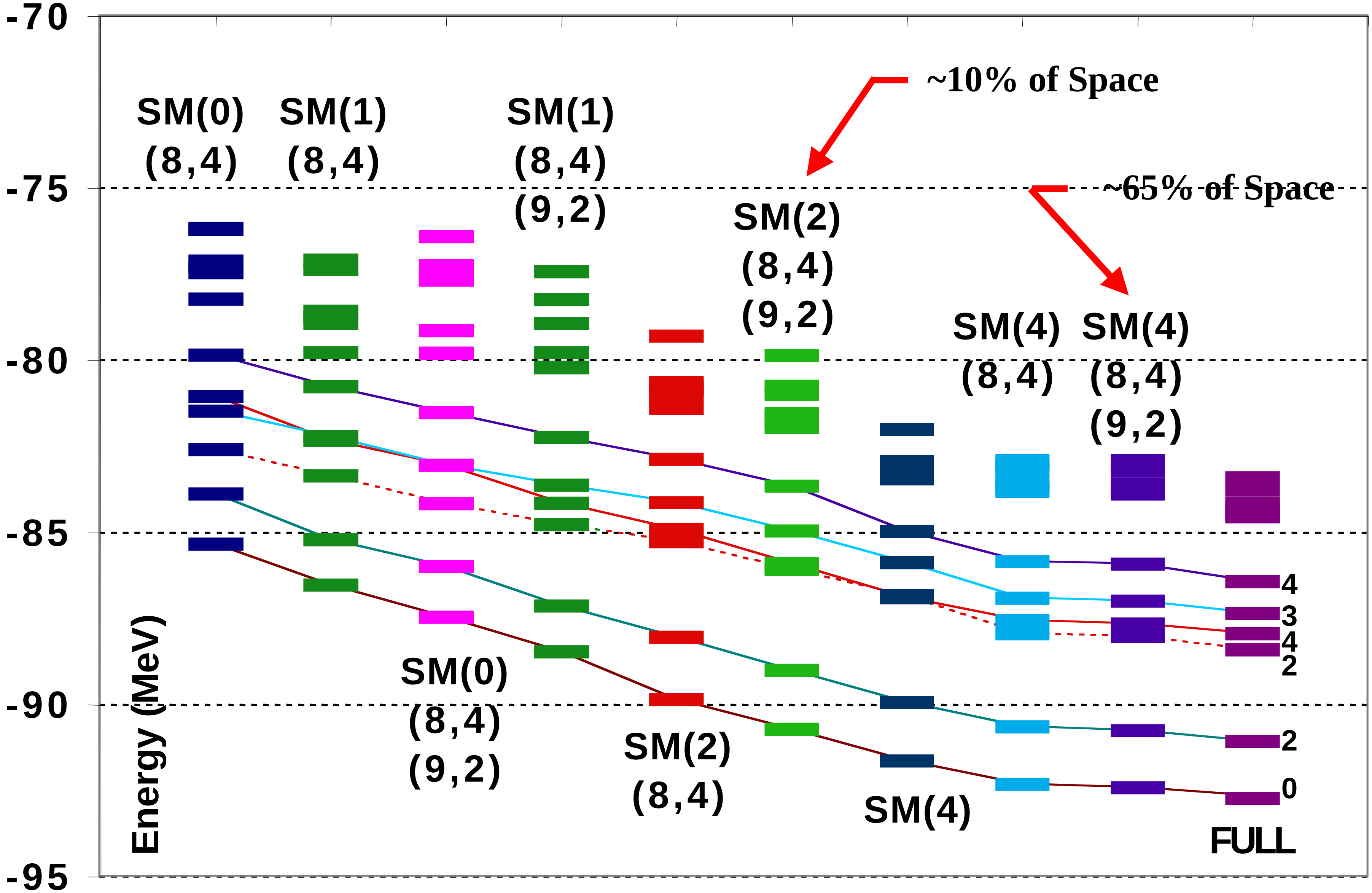
Convergence of ^{24}Mg Ground State



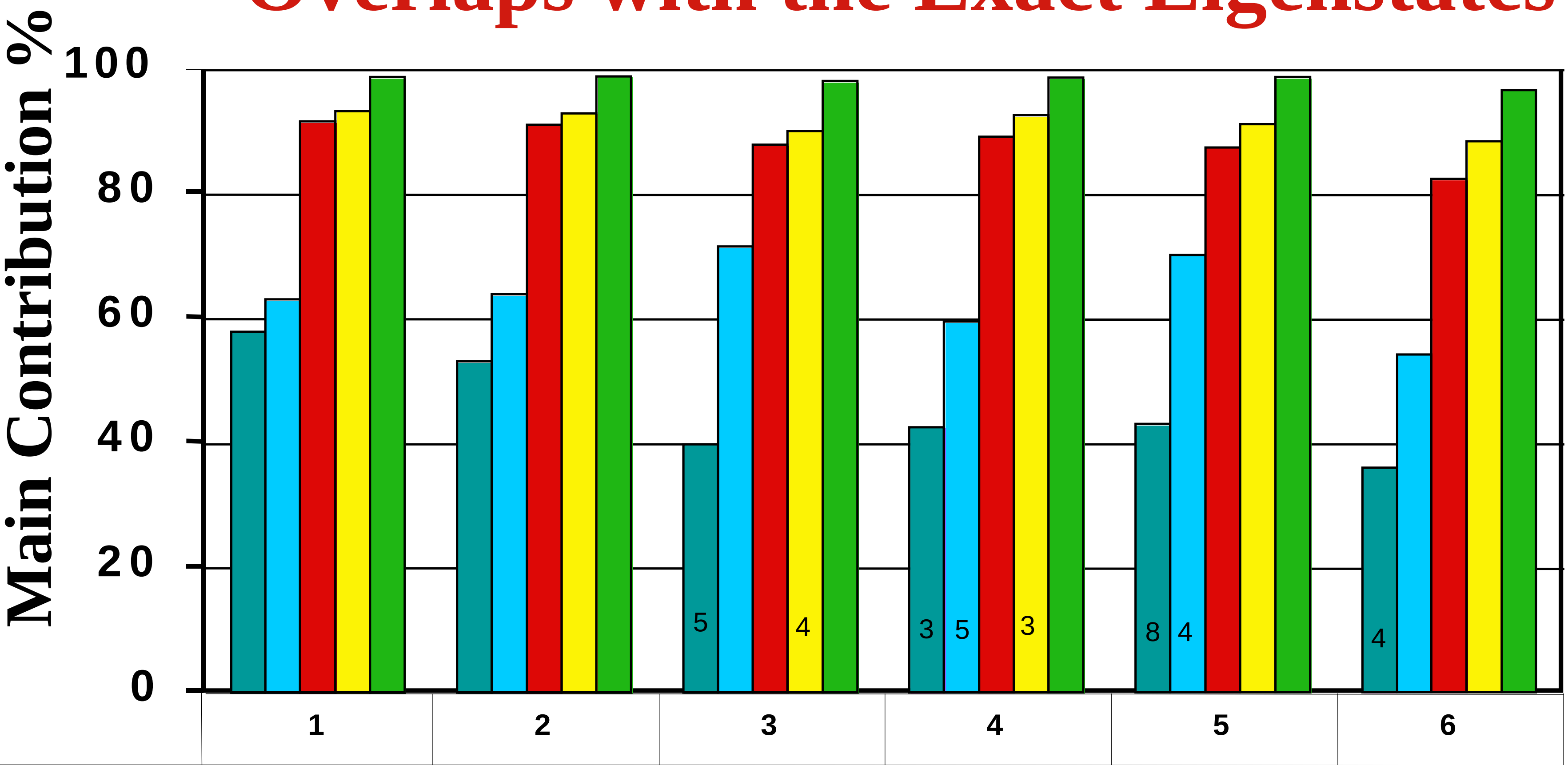
^{24}Mg - Level Structure



Oblique Basis Spectral Results



Overlaps with the Exact Eigenstates



	1	2	3	4	5	6
SM(2)	57.77	53.02	39.78	42.50	42.99	35.92
(8,4)	63.02	63.77	71.49	59.46	70.15	54.14
SM(2)+(8,4)&(9,2)	91.58	90.95	87.72	89.06	87.35	82.23
SM(4)	93.25	92.81	89.98	92.47	91.10	88.33
SM(4)+(8,4)&(9,2)	98.57	98.73	97.92	98.41	98.55	96.59

Eigenvectors

Summary

Use of two different sets of states *can enhance our understanding of complex systems.*

- There is better dimensional convergence.
- Correct level order of the low-lying states.
- Significant overlap with the exact states.